Comparison Analysis between DNMA Method and Other MCDM Methods

Xingli Wu, Huchang Liao*

Business School, Sichuan University, Chengdu 610064, China

Submitted: Apr. 30, 2019 — Accepted: Apr. 30, 2019 — Published: Apr. 30, 2019

Abstract—The utility value-based multiple criteria decision making methods have been widely used in practice. They are simple in calculation and easy to understand. The normalization and aggregation are main parts of the utility value-based methods. There are mainly two normalization techniques with different advantages, including the linear normalization and vector normalization, and three types of aggregation approaches with different functions, including the complete compensatory operator, the un-compensatory operator and the incomplete compensatory operator. The double normalization-based multiple aggregation (DNMA) method, as a new member of the utility value-based methods, has taken the advantages of both two normalization techniques and three aggregation approaches. This paper aims to make a comparative analysis between the DNMA method and other representative utility value-based methods, including the TOPSIS, VIKOR and MULTIMOORA.

Keywords—Multiple criteria decision making; Utility value-based method; Double normalization-based multiple aggregation method; Comparative analysis; TOPSIS; VIKOR; MULTIMOORA

I. INTRODUCTION

PRACTICAL decision-making problems are usually failed to be solved by the measurement of a single criterion due to their complexity and ill-structure [1]. A complex decision-making problem is characterized by multiple attributes, goals, or objectives (here, we collectively call them criteria) which are conflicting in general. Multiple criteria decision making (MCDM) is an advanced direction and hot topic of operational research that aims to structure and simplify complex decision-making problems. In the 1960s, the MCDM as a normative method began to be introduced into decision-making field, which was marked by Charnes and Cooper's research on goal programming [2] and Roy's ELECTRE (ELimination Et Choix Traduisant la REalité in French, ELimination and Choice Expressing the Reality) method [3]. The discrete form of MCDM process is a selection, ranking or sorting action among a finite set of alternatives which are measured by multiple criteria. Classically, there are nearly 20 kinds of MCDM methods which tackle the problems from diverse perspectives [4-6], including AHP (Analytic Hierarchical Process) [7], PROMETHEE (Preference Ranking Organization METHod for Enrichment of Evaluations) [8], TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) [4, 9], VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje, meaning in Serbian, meaning multiple criteria optimization compromise solution in English) [10-12], ORESTE (organísation, rangement et Synthèse de données relarionnelles, in French) [13, 14], and ELECTRE [15, 16]. In recent years, some new MCDM methods have been developed such as MULTIMOORA (Multiplicative Multi-Objective Optimization by Ratio Analysis) [17-21], BWM (Best West Method) [22, 23], GLDS (Gained and Lost Dominance Score) [24, 25] and DNMA (Double Normalization-based Multiple Aggregation) [26, 27]. The MCDM can be categorized into three groups [4-6], including the utility value-based methods, such as the TOPSIS, VIKOR, MULTIMOORA, DNMA, the outranking methods, such as the PROMETHEE, ELECTRE, GLDS, and preference ordering-based methods, such as the AHP and BWM.

The utility value-based methods have the advantages of the simple calculation, easy to understand, and available of ranking set. These merits make them become the most widely used MCDM methods in practical applications. The basic assumption of the utility value-based MCDM methods is that there is a utility function for each criterion c_j ($j=1,2,\cdots,n$). The utility values are dimensionless. Aggregation functions are applied to integrate the utility values of each alternative on all criteria into a collective one. Then, the simple rank-ordering of alternatives can be determined by the preference relation of these collective utility values. It requires the underlying condition that the criteria are mutually independent.

There are three representative approaches of the utility value-based MCDM methods, i.e., the TOPSIS, VIKOR and MULTIMOORA. They are different from the normalization and aggregation techniques. It is known that the normalization techniques have different advantages and disadvantages [28], and the aggregation techniques have different functions [27]. However, the classical utility value-based methods apply only one normalization technique, which would mislead the

* Corresponding Author: Huchang Liao, *PhD*

E-mail: liaohuchang@scu.edu.cn

This project was supported by the National Natural Science Foundation of China, Grant No. 71771156, the 2019 Sichuan Planning Project of Social Science, Grant No. SC18A007, the 2019 Soft Science Project of Sichuan Science and Technology Department, Grant No. 2019JDR0141, and the Project of Innovation at Sichuan University, Grant No. 2018hhs-43.

decision matrix and limit their applications. To avoid these limitations, Liao and Wu [27] proposed a new utility value-based methods, namely the DNMA method. It takes advantages of different normalization techniques and aggregation functions which are combined in an appropriate way.

This paper introduces the theory of the utility value-based methods in depth. We aim to highlight the advantages of the DNMA method by comparative analyses.

II. Different MCDM Methods

This section introduces the theory of the TOPSIS method, VIKOR method, MULTIMOORA method and DNMA method. All these four methods are based on the same decision matrix expressed as $D=[x_{ij}]_{m\times n}$ where x_{ij} is the performance value of alternative a_i ($i=1,2,\cdots,m$) with respect to criterion c_j ($j=1,2,\cdots,n$). There is a weight vector, $W=\{\omega_1,\omega_2,\cdots,\omega_n\}^T$, of criteria to denote their relative importance and $\sum_{j=1}^{n}\omega_j=1$. It represents both criterion importance from decision-makers' subjective preference and measurement scale (''trade-offs'' weights to eliminate criterion units). Given this, there are three categories of weighting methods [20]: (1) subjective weight methods, such as AHP; (2) objective weight methods, such as entropy method; (3) combined weight methods. The principle and process for solving the decision matrix are different to different MCDM methods.

A. The TOPSIS method

TOPSIS method proposed by Chen and Hwang [4] aims to determine the optimal alternative which is closest to the positive ideal solution and farthest way from the negative ideal solution. Its principle is based on the distance measure. It has simple calculation process which is shown as follows:

Step 1. Normalize the performance values by Eq. (1) and obtain the normalized decision matrix as $D^N=[x_{ij}^N]_{m\times n}$.

$$x_{ij}^N=\frac{x_{ij}}{\sqrt{\sum_{i=1}^{m}x_{ij}^2}} \tag{1}$$

Step 2. Find the positive ideal solution $A^+=\{x_1^+,x_2^+,\cdots,x_n^+\}$, and negative ideal solution $A^-=\{x_1^-,x_2^-,\cdots,x_n^-\}$, respectively. They can be determined by

$$x_j^+=\begin{cases}\max\limits_{i=1,2,\cdots,m} x_{ij}^N, \text{for benefit criterion } c_j\\ \min\limits_{i=1,2,\cdots,m} x_{ij}^N, \text{for cost criterion } c_j\end{cases} \tag{2}$$

$$x_j^-=\begin{cases}\min\limits_{i=1,2,\cdots,m} x_{ij}^N, \text{for benefit criterion } c_j\\ \max\limits_{i=1,2,\cdots,m} x_{ij}^N, \text{for cost criterion } c_j\end{cases} \tag{3}$$

Step 3. Compute the distance between each alternative and the positive ideal solution over each criterion by

$$D_i^+=\sqrt{\sum_{j=1}^{n}\left(\omega_j x_{ij}^N-\omega_j x_j^+\right)^2} \tag{4}$$

Similarly, calculate the distance between each alternative and the native ideal solution over each criterion by

$$D_i^-=\sqrt{\sum_{j=1}^{n}\left(\omega_j x_{ij}^N-\omega_j x_j^-\right)^2} \tag{5}$$

Step 4. Compute the relative closeness of each alternative to the ideal solution by Eq. (6).

$$RC(a_i)=D_i^-/(D_i^++D_i^-) \tag{6}$$

Step 5. Rank alternatives by $RC(a_i)$, $i=1,2,\cdots,m$. The larger the value $RC(a_i)$ is, the better the alternative a_i should be.

B. The VIKOR method

The VIKOR was originally proposed by Opricovic [10] to determine the compromise alternatives in the presence of conflicting criteria. Its main idea is to measure the "group utility" for the "majority" and the "individual regret" for the "opponent" of each alternative based on the distance measure. The process of the VIKOR is as follows:

Step 1. Find the best value under each criterion such that $x_j'^+=\max\limits_i x_{ij}$ for benefit criteria and $x_j'^+=\min\limits_i x_{ij}$ for cost criteria. Similarly, determine the worst value under each criterion that $x_j'^-=\min\limits_i x_{ij}$ for benefit criterion and $x_j'^-=\max\limits_i x_{ij}$ for cost criterion.

Step 2. Calculate the group utility measure and the individual regret measure of each alternative by Eq. (7) and Eq. (8), respectively.

$$GU_i=\sum_{j=1}^{m}\omega_j\frac{x_j'^+-x_{ij}}{x_j'^+-x_{ij}'^-} \tag{7}$$

$$IR_i=\max_j\left(\omega_j\frac{x_j'^+-x_{ij}}{x_j'^+-x_{ij}'^-}\right) \tag{8}$$

Step 3. Compute the compromise measure of each alternative by

$$C_i=\lambda\frac{GU_i-GU^+}{GU^--GU^+}+(1-\lambda)\frac{IR_i-IR^+}{IR^--IR^+} \tag{9}$$

where $GU^+=\min\limits_i GU_i$, $GU^-=\max\limits_i GU_i$, $IR^+=\min\limits_i IR_i$ and $IR^-=\max\limits_i IR_i$, and $\lambda\in[0,1]$ is a parameter to denote the relative weight of the group utility measure and the individual regret measure.

Step 4. The ranking set can be determined by GU_i, IR_i and C_i together. The compromise principles are:

Condition 1. Acceptable advantage: $C_{a_i''}-C_{a'}\geq 1/(n-1)$, where a'' is ranked at second according to the value C_i;

Condition 2. Acceptable stability: the alternative a' is ranked at first by both GU_i and IR_i.

There are more than one compromise solution if one of the two conditions are not satisfied:

(i) Alternatives a' and a'' if only Condition 2 is not satisfied;

(ii) Alternatives $a', a'', \cdots, a^{(m)}$ if Condition 1 is not satisfied, where $a^{(m)}$ is established by the relation $C_{a^{(m)}} - C_{a'} < 1/(n-1)$ for the maximum m.

C. The MULTIMOORA method

The MULTIMOORA method was developed by Brauers and Zavadskas [18] with reference to the MOORA method [17]. It is characterized by three kinds of aggregation approaches to derive three kinds of collective utility values of each alternative. Finally, the dominance theory [29] is applied to integrate these three subordinate utility values. The detail steps of the MULTIMOORA method are shown below.

Step 1. Like the Step 1 of the TOPSIS method to build the normalize decision matrix $D^N = [x_{ij}^N]_{m\times n}$ by Eq. (1).

Step 2. Compute three kinds of subordinate utility values of each alternative by the ratio system as Eq. (10), the reference point approach as Eq. (11) and the full multiplicative form as Eq. (12), respectively.

$$U_1(a_i) = \sum_{j=1}^{g} \omega_j x_{ij}^N - \sum_{j=g+1}^{n} \omega_j x_{ij}^N \tag{10}$$

$$U_2(a_i) = \max_j \omega_j \left| r_j - x_{ij}^N \right| \tag{11}$$

$$U_3(a_i) = \Pi_{j=1}^{g} \left(x_{ij}^N\right)^{\omega_j} \Big/ \Pi_{j=g+1}^{n} \left(x_{ij}^N\right)^{\omega_j} \tag{12}$$

where $\{c_j \mid j = 1,2,\cdots,g\}$ are the benefit criteria and $\{c_j \mid j = g+1, g+2, \cdots, n\}$ are the cost criteria.

Step 3. The ranking set should be determined by $U_1(a_i)$, $U_2(a_i)$ and $U_3(a_i)$ together with the help of the dominance theory. The larger the values $U_1(a_i)$ and $U_3(a_i)$ are, the better the alternative a_i is, and the smaller the value of $U_2(a_i)$ is, the better the alternative a_i should be.

D. The DNMA method

The DNMA method [26, 27] is to rank alternatives of complex MCDM problems with conflicting criteria. The basic principle is that the chosen alternative is the closest to the expectation solution which consists of the expectation value of each criterion. The expectation values of criteria may be the maximum one, the minimum one or the medium one. The DNMA method focuses on taking the advantages of different normalization techniques and aggregation functions, and makes an appropriate combination of the normalization and aggregation approaches. The steps of the DNMA method are shown as follows:

Step 1. Normalize the performance values by Eq. (13) and Eq. (14), respectively. Then obtain two normalized decision matrices $\tilde{D}^{1N} = [\tilde{x}_{ij}^{1N}]_{m\times n}$ and $\tilde{D}^{2N} = [\tilde{x}_{ij}^{2N}]_{m\times n}$.

$$\tilde{x}_{ij}^{1N} = 1 - \frac{\left|x^{ij} - r_j\right|}{\max\left\{\max_i x^{ij}, r_j\right\} - \min\left\{\min_i x^{ij}, r_j\right\}} \tag{13}$$

$$\tilde{x}_{ij}^{2N} = 1 - \frac{\left|x^{ij} - r_j\right|}{\sqrt{\sum_{i=1}^{m}\left(x^{ij}\right)^2 + \left(r_j\right)^2}} \tag{14}$$

where r_j is the expectation value of criterion c_j. Especially, $r_j = \max_i x_{ij}$ for the benefit criterion c_j and $r_j = \min_i x_{ij}$ for the cost criterion c_j.

Step 2. Adjust the criterion weight to make a trade-off between criteria after the normalization. The weight adjustment coefficient of criterion c_j is computed by

$$\omega_j^\sigma = \sigma_j \Big/ \sum_{j=1}^{n} \sigma_j \tag{15}$$

where σ_j is the standard deviation of criterion c_j such that

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^{m}\left(x^{ij}/\max_i x^{ij} - \frac{1}{m}\sum_{i=1}^{m}\left(x^{ij}/\max_i x^{ij}\right)\right)^2}{m}} \tag{16}$$

The criterion weights are adjusted by

$$\tilde{\omega}_j = \sqrt{\omega_j^\sigma \cdot \omega_j} \Big/ \sum_{j=1}^{n} \sqrt{\omega_j^\sigma \cdot \omega_j} \tag{17}$$

Step 3. Compute three kinds of subordinate utility values by the complete compensatory model (CCM) as Eq. (18), the un-compensatory model (UCM) as Eq. (19) and the incomplete compensatory model (ICM) as Eq. (20), respectively.

$$u_1(a_i) = \sum_{j=1}^{n} \tilde{\omega}_j \tilde{x}_{ij}^{1N} \Big/ \max_i \tilde{x}_{ij}^{1N} \tag{18}$$

$$u_2(a_i) = \max_j \tilde{\omega}_j (1 - \tilde{x}_{ij}^{1N} \Big/ \max_i \tilde{x}_{ij}^{1N}) \tag{19}$$

$$u_3(a_i) = \prod_j (\tilde{x}_{ij}^{2N} \Big/ \max_i \tilde{x}_{ij}^{2N})^{\omega_j} \tag{20}$$

Step 4. Integrate the three types of subordinate utility values of each alternative by the weighted Euclidean distance formula as

$$\begin{aligned} DN_i = {} & w_1 \sqrt{\varphi\left(u_1(a_i)/\max_i u_1(a_i)\right)^2 + (1-\varphi)\left(\frac{m - r_1(a_i) + 1}{m}\right)^2} \\ & - w_2 \sqrt{\varphi\left(u_2(a_i)/\max_i u_2(a_i)\right)^2 + (1-\varphi)\left(\frac{r_2(a_i)}{m}\right)^2} \\ & + w_3 \sqrt{\varphi\left(u_3(a_i)/\max_i u_3(a_i)\right)^2 + (1-\varphi)\left(\frac{m - r_3(a_i) + 1}{m}\right)^2} \end{aligned} \tag{21}$$

where $r_1(a_i)$ and $r_3(a_i)$ are the ranks of alternative a_i determined by the descending order of $u_1(a_i)$ and $u_3(a_i)$, respectively, $r_2(a_i)$ is the rank of alternative a_i determined by the ascending order of $u_2(a_i)$, φ is a parameter to denote the relative importance of the utility values and ranks that $\varphi \in [0,1]$, and w_1, w_2, w_3 are the weights of three aggregation models, satisfying $w_1 + w_2 + w_3 = 1$.

Step 5. The ranking set is determined in descending order DN_i, $i = 1,2,\cdots,m$.

III. Comparative Analyses

The section compares the DNMA method with the TOPSIS, the VIKOR and MULTIMOORA theoretically in terms of the normalization approach, aggregation function and integration method.

A. Normalization approach

Normalization as a dimensionality reduction process is critical for the MCDM since we can make aggregation only when all criteria are dimensionless. There are mainly two categories of normalization techniques which are widely used in solving MCDM problems, including the linear normalization and the vector normalization.

The linear normalization has been used in the VIKOR method [10], the extended TOPSIS method [21] and the DNMA method [27]. Its general form can be defined as:

$$N(x_{ij}) = \frac{\max_i x_{ij} - x_{ij}}{\max_i x_{ij} - \min_i x_{ij}} \tag{22}$$

The performance values of alternatives over each criterion are normalized to 0 and 1 linearly. They are scaled up or down according to the possible range of values of each criterion. We can find that the proportion of the value compared with the maximum value in term of the geometric relationship is not changed after linear normalization. In addition, we can obtain the same normalization value for different convertible units of a criterion. However, the scale of the value itself compared with the scale of whole values may be changed. What's more, for each criterion, there always exists a normalized value 0.

The vector normalization has been used in the classical TOPSIS method, the MULTIMOORA method and the DNMA method. The general form of the vector normalization is as Eq. (1). The performance values are normalized to 0 and 1 by comparing with the sum of a set of values nonlinearly. The proportion of the original value in the whole is changed, but its scale compared with the size of whole values is not changed. However, we may obtain different vector normalization values for the criterion with different convertible units.

As illustrated by Liao and Wu [27], both the linear normalization and vector normalization have their own advantages and disadvantages. The linear normalization can preserve its proportion and unique value for different convertible units of a criterion, but it loses the original scale compared with the whole values. Nonetheless, the vector normalization has the opposite characteristics. That is, the strengths and weaknesses of the linear normalization and vector normalization complement each other. They are applicable to different types of MCDM problems. Except the DNMA method, all other MCDM methods use only one normalization approaches to nondimensionalize the decision matrix. Using one type of normalization would increase the risk of information distortion since inappropriate techniques may be used.

B. Aggregation function

Usually, the normalization function $N(x_{ij})$ acts as the utility function. The aggregation functions are mainly organized by the additive operation (as Eq. (23) and Eq. (24)) or the multiplication operation (as Eq. (25)).

(a) The additive form is:

$$U^1(x_{i1}, x_{i2}, \cdots, x_{in}) = \left\{ \sum_{j=1}^{n} \left(\omega_j N(x_{ij}) \right)^p \right\}^{1/p} \tag{23}$$

or

$$U'^1(x_{i1}, x_{i2}, \cdots, x_{in}) = \left\{ \sum_{j=1}^{n} \left(1 - \omega_j N(x_{ij}) \right)^p \right\}^{1/p} \tag{24}$$

where p is a parameter with $1 \le p \le \infty$.

Note. If $p = 1$ (for Eq. (23) and Eq. (24)), the weighted value $\omega_j N(x_{ij})$ ($j = 1, 2, \cdots, n$) of each criterion is equally important. For Eq. (23), if $p = 2$, the larger the weighted value is, the larger the importance of it would be. With the increase of p, the potential importance of the larger weighted value increases. Especially, the greatest value $\max_i \omega_j N(x_{ij})$ is the dominant element when $p = \infty$ such that $U^1 = \max_i \omega_j N(x_{ij})$.

The aggregation operator U^1 of $p = 1$ has been applied in the VIKOR method, MULTIMOORA method and DNMA method. This is a complete compensatory operator for aggregating the performances of alternatives in MCDM such that the bad performances of an alternative over some criteria can be completely compensated by the good performances over other criteria.

The aggregation operator U^1 with $p = 2$ has been used in the TOPSIS method. This is an incomplete compensatory operator such that the good performances play the dominance relation in the operator. This operator implies that the decision-maker is inclined to measure the good side of alternatives when making decision.

The aggregation operator U'^1 has been used in the VIKOR method, MULTIMOORA method and DNMA method. This is an un-compensatory operator for aggregating the dimensionless performances of alternatives. This operator focuses on the worst performance of an alternative which cannot be compensated by the good performances of this alternative under some criteria.

(b) The multiplication form is:

$$U^2(x_{i1}, x_{i2}, \cdots, x_{in}) = \prod_{j=1}^{n} \left(N(x_{ij}) \right)^{w_j} \tag{25}$$

There is an additional requirement that $N(x_{ij}) \ne 0$; otherwise, the equation is invalid. This is another kind of incomplete compensatory operator. Unlike the aggregation operator U^1 with $p = 2$, Eq. (25) inclines to measure the bad side of alternatives. The bad performances of an alternative under some criteria cannot be completely compensated by the good performances of this alternative under other criteria. The poor performance over some criteria can be a drag on the overall performance. For example, if there are two alternatives and two criteria. One alternative performs moderately under two criteria, and another alternative performs very good under one criterion and very bad under another criterion. By the complete compensatory operator as U^1 with $p=1$, we obtain that these two alternatives have the same collective performance. Then, by Eq. (25), we can derive that the first alternative is superior to the second one. This property meets some practical requirements of decision making problems in which decision-makers are reluctant to take risks in choosing solutions that perform poorly under some criteria.

The TOPSIS method combines the vector normalization with the incomplete compensatory operator (U^1 with $p=2$) to construct the aggregation model. Some researchers tend to use the linear normalization instead of the vector form when using the TOPSIS.

The VIKOR method integrates the linear normalization with the complete compensatory operator (U^1 of $p=1$) and the un-compensatory operator (U'^1 of $p=\infty$), to construct the group utility measure and the individual regret measure, respectively. Its main idea is that the selected solution should has a large collective utility value (this is reflected by the group utility value) and is not very bad under all criteria (this is reflected by the individual regret value).

The MULTIMOORA method combines the vector normalization with the complete compensatory operator (U^1 with $p=1$), the un-compensatory operator (U'^1 with $p=\infty$) and the incomplete compensatory operator (U^2) to establish three kinds of aggregation models, respectively. Its aims to find the best solution which are superior to others considering the trade-off between three utility values.

The DNMA method combines the linear normalization with the complete compensatory operator (U^1 with $p=1$), the un-compensatory operator (U'^1 with $p=\infty$). The reason for this combination but not using the vector normalization is that the linear normalization can preserve the proportion of original values compared with the whole values. This is coincidence with the distance measure to the ideal solution.

The third aggregation model of the DNMA method is constructed by the vector normalization with the incomplete compensatory operator (U^2). The reason for this combination is that the incomplete compensatory operator has the property which is closer to the risk attitude of decision-makers. This property is not existed in other aggregation operators mentioned above. However, it requires $N(x_{ij})\neq 0$. Thus, the linear normalization is not suitable to combine with the operator U^2. But the vector normalization is appropriate. In this way, both the advantages of two normalization techniques and the functions of three aggregation operators are brought into play.

C. *Integration method*

In the final process of the utility value-based MCDM methods, it requires to integrate the utility values derived by different aggregation operators for each alternative to determine their ranking set.

The TOPSIS method uses only one aggregation operator (U^1 with $p=2$) to compute the closeness degree to the positive idea solution and the negative idea solution, respectively. These two kinds of closeness degrees are aggregated by a ratio operator as (Eq. (6)). However, the relative importance which is a major concern for decision-making is ignored by Eq. (6). Opricovic and Tzeng [30] illustrated that the selected solution by the TOPSIS method may be not closest to the ideal solution.

The VIKOR method uses an additive weighted operator (as Eq. (9)) to integrate the group utility values and the individual regret values of each alternative. The derived compromise values are sensitive to the relative importance of the two kinds of utility values. This has been illustrated in many studies [20, 27, 31]. This increases the difficulty of decision-makers in selecting appropriate parameter λ. In addition, the compromise principle is strict that we usually obtain many compromise solutions. In most cases, however, we only need to select one solution.

The MULTIMOORA method integrates three types of subordinate ranks, respectively, derived by three kinds of subordinate utility values to a collective one based on the dominance theory. In this way, the final rank set is difficult to be determined since pairwise comparisons are needed among alternatives [21]. Moreover, there are many confusion relations when comparing the subordinate ranks of two alternatives.

The DNMA method considers both the utility values and the subordinate ranks of alternatives to determine the collective values of alternatives by a weighted Euclidean distance formula. In this way, the sensitivity for the relative importance of the aggregation operators is reduced and we can obtain a relative robust result. In addition, the computation is simple compared with the dominance principle. Also, a strict ranking set can be derived. Liao and Wu [27] proved the advantages of considering both the utility values and ranks in the integration process compared with only considering utility values (used in the VIKOR method) or subordinate ranks (used in the MULTIMMORA method).

IV. Conclusions

This paper compared the TOPSIS, VIKOR, MULTIMOORA with the DNMA method. They are different from the normalization, aggregation and integration process. The first three methods only consider one normalization approach. This may cause biased results since each normalization approach may lead information loss to some extent. In addition, the TOPSIS can select the optimal solution which performs the best overall since the complete compensatory aggregation is used. The VIKOR can select the solution which performs well overall and not very bad under all criteria since both the complete compensatory aggregation and non-compensatory aggregation models are used. The MULTIMOORA also applies more than one aggregation model to measure different aspects of the alternatives. However, these three MCDM methods have not considered the appropriate combination between the normalization and aggregation approaches.

The DNMA method aims to combine different normalization and aggregation approaches in an appropriate manner to maximum their advantages. Compared with the TOPSIS, VIKOR and MULTIMOORA, the DNMA method has the following advantages:

1) Flexibility: We can adjust the weights of different aggregation models according to decision-making requirements and objectives as well as the decision-makers' risk attitudes.

2) Reliability: The advantages and disadvantages of the two normalization methods used in the DNMA method are compensatory. Therefore, the information loss caused in the normalization process can be reduced. Three kinds of aggregation models ensure that the DNMA can rank the alternatives by making trade-off between collective performances and worst performances.

3) Simplicity: The integration approach used in the DNMA method can derive a rank set of alternatives directly.

The paper does not make the practical applications to illustrate the comparative analysis. This is an important future work.

REFERENCES

[1] C. Kahraman, S. C. Onar, and B. Oztaysi, "Fuzzy multicriteria decision-making: a literature review," *International Journal of Computational Intelligence Systems*, vol. 8, no. 4, pp. 637-666, 2015.

[2] F. Glovera, and T. Sueyoshi, "Contributions of professor William W. Cooper in operations research and management science," *European Journal of Operational Research*, vol. 197, no. 1, pp. 1-16, 2009.

[3] B. Roy, "Classement et choix en presence de points de vue multiples (La methode ELECTRE)," *Revue Francaise D Informatique de Recherche Operationnelle*, vol. 2, no. 8, pp. 57-75, 1968.

[4] S. J. Chen, and C. L. Hwang, "Fuzzy multiple attribute decision making: methods and applications," SpringerVerlag, Berlin, 1992.

[5] C. Kahraman, S. C. Onar, and B. Oztaysi, "Fuzzy multicriteria decision-making: a literature review," *International Journal of Computational Intelligence Systems*, vol. 8, no. 4, pp. 637-666, 2015.

[6] H. C. Liao, Z. S. Xu, E. Herrera-Viedma, and F. Herrera, "Hesitant fuzzy linguistic term set and its application in decision making: a state-of-the art survey," *International Journal of Fuzzy Systems* vol. 20, no. 7, pp. 2084-2110, 2018.

[7] Z. S. Xu, and H. C. Liao, "Intuitionistic fuzzy analytic hierarchy process," *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 4, pp. 749-761, 2014.

[8] H. C. Liao, and Z. S. Xu, "Multi-criteria decision making with intuitionistic fuzzy PROMETHEE," *Journal of Intelligent & Fuzzy Systems*, vol. 27, no.4, pp. 1703-1717, 2014.

[9] C. T. Chen, "Extensions of the TOPSIS for group decision-making under fuzzy environment," *Fuzzy Sets and Systems*, vol, 114, no. 1, pp. 1-9, 2000.

[10] S. Opricovic, *Multicriteria Optimization of Civil Engineering Systems*, Faculty of Civil Engineering, Belgrade, 1998.

[11] H. C. Liao, and Z. S. Xu, "A VIKOR-based method for hesitant fuzzy multi-criteria decision making," *Fuzzy Optimization and Decision Making*, vol. 12, no. 4, pp. 373-392, 2013.

[12] H. C. Liao, Z. S. Xu, and X.J. Zeng, "Hesitant fuzzy linguistic VIKOR method and its application in qualitative multiple criteria decision making," *IEEE Transactions on Fuzzy Systems*, vol. 23, no. 5, pp. 1343-1355, 2015.

[13] X. L. Wu, and H. C. Liao, "An approach to quality function deployment based on probabilistic linguistic term sets and ORESTE method for multi-expert multi-criteria decision making," *Information Fusion*, vol. 43, pp. 13-26, 2018.

[14] H. C. Liao, X. L. Wu, X. D. Liang, J. B. Yang, D. L. Xu, and F. Herrera, "A continuous interval-valued linguistic ORESTE method for multi-criteria group decision making," *Knowledge-Based Systems*, vol. 153, pp. 65-77, 2018.

[15] H. C. Liao, L. Y. Yang, and Z. S. Xu, "Two new approaches based on ELECTRE II to solve the multiple criteria decision making problems with hesitant fuzzy linguistic term sets," *Applied Soft Computing*, vol. 63, pp. 223-234, 2018.

[16] H. C. Liao, X. L. Wu, X. M. Mi, and F. Herrera, "An integrated method for cognitive complex multiple experts multiple criteria decision making based on ELECTRE III with weighted Borda rule," *Omega*, in press, 2019. DOI: 10.1016/j.omega.2019.03.010.

[17] W. K. M Brauers, and E. K. Zavadskas, "The MOORA method and its application to privatization in a transition economy," *Control and Cybernetics*, vol. 35, no. 2, pp. 445-469, 2006.

[18] W. K. M. Brauers, and E. K. Zavadskas, "Project management by multimoora as an instrument for transition economies," *Technological and Economic Development of Economy*, vol. 16, no. 1, pp. 5-24, 2010.

[19] A. Hafezalkotob, A. Hafezalkotob, H. C. Liao, and F. Herrera, "An overview of MULTIMOORA for multi-criteria decision-making: theory, developments, applications, and challenges," Information Fusion, vol. 51, 145-177, 2019.

[20] X. L. Wu, H. C. Liao, Z. S. Xu, A. Hafezalkotob, and F. Herrera, "Probabilistic linguistic MULTIMOORA: a multi-criteria decision making method based on the probabilistic linguistic expectation function and the improved Borda rule," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 6, pp. 3688-3702, 2018.

[21] H. C. Liao, R. Qin, C. Y. Gao, X. L. Wu, A. Hafezalkotob, and F. Herrera, "Score-HeDLiSF: a score function of hesitant fuzzy linguistic term set based on hesitant degrees and linguistic scale functions: an application to unbalanced hesitant fuzzy linguistic MULTIMOORA," *Information Fusion*, vol. 48, pp. 39-54, 2019.

[22] J. Rezaei, "Best-worst multi-criteria decision-making method," *Omega*, vol. 53, pp. 49-57, 2015.

[23] X. M. Mi, M. Tang, H. C. Liao, W. J. Shen, and B. Lev, "The state-of-the-art survey on integrations and applications of the best worst method in decision making: Why, what, what for and what's next?" Omega, in press, 2019. DOI: 10.1016/j.omega.2019.01.009

[24] X. L. Wu, and H. C. Liao, "A consensus based probabilistic linguistic gained and lost dominance sore method," *European Journal of Operational Research*, vol. 272, no. 3, pp. 1017-1027, 2019.

[25] Z. G. Fu, X. L. Wu, H. C. Liao, and F. Herrera, "Underground mining method selection with the hesitant fuzzy linguistic gained

and lost dominance score method," *IEEE Access*, vol. 6, no. 1, pp. 66442-66458, 2018.

[26] H. C. Liao, X. L. Wu, and F. Herrera, "DNBMA: a double normalization-based multi-aggregation method," Information Processing and Management of Uncertainty in Knowledge-Based Systems Applications, IPMU, 2018, Springer, Cham, 2018, pp. 63–73.

[27] H. C. Liao, and X. L. Wu, "DNMA: a double normalization-based multiple aggregation method for multi-expert multi-criteria decision making," *Omega*, in press, 2019. DOI: 10.1016/j.omega.2019.04.001.

[28] A. Jahan, and K. L. Edwards, "A state-of-the-art survey on the influence of normalization techniques in ranking: improving the materials selection process in engineering design," *Materials & Design*, vol. 65, pp. 335-342, 2015.

[29] W. K. M. Brauers, A. Baležentis, and T. Baležentis, "Multimoora for the EU member states updated with fuzzy number theory," Technological and Economic Development of Economy, vol. 17, no. 2, pp. 259–290, 2011.

[30] S. Opricovic and G. H. Tzeng, "Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS," *European Journal of Operational Research*, vol. 156, pp. 445-455, 2004.

[31] H. C. Liao, Z. S. Xu, and X. J. Zeng, "Distance and similarity measures for hesitant fuzzy linguistic term sets and their application in multi-criteria decision making," *Information Sciences*, vol. 271, pp. 125-142, 2014.

Xingli Wu is obtained her Master degree in Industrial Engineering at Sichuan University, Sichuan, China in 2019. She has published 15 peer-reviewed papers, many in high-quality international journals including European Journal of Operational Research, Omega, IEEE Transactions on Fuzzy Systems, Information Fusion, Knowledge-Based Systems, etc. She was a recipient of the National Encouragement scholarship for undergraduates of China Awards both in 2014 and 2015 and National scholarship for graduates of China Awards in 2018. She won the First Prize Scholarship for graduates of SCU Awards in 2016 and 2018 and Baosteel Scholarship in 2018. She is the "Top Ten Academic Stars" of graduate student of Sichuan University. Her current research interests include group decision making, information fusion, and multiple criteria decision making under uncertainty.

Huchang Liao is a Research Fellow in the Business School at the Sichuan University, Chengdu, China. He received his PhD degree in Management Science and Engineering from the Shanghai Jiao Tong University, Shanghai, China, in 2015. He was an Academic Visitor at the University of Manchester, UK, an Endeavour Research Fellow at the University of Technology Sydney, Australia, an Invited Professor at the University of Granada, Spain. He has published 3 monographs, 1 chapter, and more than 140 peer-reviewed papers, many in high-quality international journals including European Journal of Operational Research, Omega, IEEE Transactions on Fuzzy Systems, IEEE Transaction on Cybernetics, Information Sciences, Information Fusion, Knowledge-Based Systems, Fuzzy Sets and Systems, Expert Systems with Applications, International Journal of Production Economics, etc. He was ranked top 702 in the ESI top 1% of the world's 3354 most influential scientists in the field of Computer Science, and top 4926 in the ESI top 1% of the world's 8993 most influential scientists in the field of Engineering, in March 2019. His current research interests include multiple criteria decision analysis under uncertainty, business intelligence and data science, cognitive computing, fuzzy set and systems, healthcare management, evidential reasoning theory with applications in big data analytics, etc. Prof Liao is the Senior Member of IEEE since 2017. He is the Editor-In-Chief, Associate Editor, Guest Editor or Editorial Board Member for 30 international journals, including Information Fusion (SCI, impact factor: 6.639), Technological and Economic Development of Economy (SSCI, impact factor: 3.244), Computers & Industrial Engineering (SCI, impact factor: 3.195), International Journal of Fuzzy Systems (SCI, impact factor: 2.396), Journal of Intelligent & Fuzzy Systems (SCI, impact factor: 1.426) and Mathematical Problems in Engineering (SCI, impact factor: 1.145). Prof Liao has received numerous honors and awards, including the Thousand Talents Plan for Young Professionals in Sichuan Province, the Candidate of Academic and Technical Leaders in Sichuan Province, the 2017 Outstanding scientific research achievement award in higher institutions (First class in Natural Science), the 2019 Outstanding scientific science research achievement award in Sichuan Province (Second class in Social Science), the 2016 Dongtu Academic Innovation Award granted by the Sichuan University, the 2015 Endeavour Research Fellowship award granted by the Australia Government.